RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

FIRST YEAR [BATCH 2017-20]
B.A./B.Sc. SECOND SEMESTER (January – June) 2018
Mid-Semester Examination, March 2018

Date : 14/03/2018 **PHYSICS (Honours)**
Time : 11 am – 1 pm Paper : II Full Marks : 50

Answer <u>any five</u> questions taking at least <u>one from each group</u> [5×10]

<u>Group – A</u>

1. a) What do you mean by a basis of a vector space? Explain with example. [3]

   b) If α, β, γ be a basis of a vector space in $\mathbb{R}^3$, then show that $\alpha+\beta, \beta+\gamma, \gamma+\alpha$ is also a set of basis in $\mathbb{R}^3$, where $\mathbb{R}^3$ denotes the three dimensional real space. [2]

   c) Show that $\delta_{ij} \in_{klm}$ is an isotropic tensor of rank 5. [2]

   d) Prove the Lagrange's identity : $(\vec{A}\times\vec{B})\cdot(\vec{C}\times\vec{D}) = (\vec{A}\cdot\vec{C})(\vec{B}\cdot\vec{D}) - (\vec{A}\cdot\vec{D})(\vec{B}\cdot\vec{C})$ using tensor notation. [3]

2. a) A linear operator A is defined by $A\hat{i} = \hat{j}$, $A\hat{j} = \hat{k}$ and $A\hat{k} = \hat{i}$ where $\hat{i}, \hat{j}$ and $\hat{k}$ are the unit vectors of the Cartesian coordinates. Find the matrix representation of A. [2]

   b) What are the eigenvalues and eigenvectors of A? [1+3]

   c) What is the matrix representation of A in the basis $\hat{i}' = \frac{\hat{i}+\hat{j}}{\sqrt{2}}$, $\hat{j}' = \frac{\hat{i}-\hat{j}}{\sqrt{2}}$, $\hat{k}' = \hat{k}$. [3]

   d) What is the transformation matrix S that takes one from the first Cartesian coordinate frame to the primed one? [1]

<u>Group – B</u>

3. a) Show that for a system of particles at rest the location of the centre-of-mass (CM) is independent of the choice of the origin of the reference frame. [2]

   b) Consider a two-particle system rotating about an arbitrary but given axis through the CM with angular velocity $\vec{\omega}$. Show that the angular momentum about the CM is, in general, not parallel to $\vec{\omega}$. For what choice of the axis are they parallel? [4]

   c) Show that the kinetic energy (K.E.) of a many-particle system can be expressed as a sum of the K.E. of a fictitious particle at the CM and the K.E. of the system relative to the CM. [4]

4. a) A vector $\vec{A}(t)$ of fixed magnitude rotates about a given axis with angular velocity $\vec{\omega}$. Show that the time rate of change of $\vec{A}$ is given by $\frac{d\vec{A}}{dt} = \vec{\omega}\times\vec{A}$. [4]

   b) Let S be a space-fixed Cartesian frame, and R a rotating frame having a common origin, O. The velocity of a moving particle with instantaneous position vector $\vec{r}(t)$ is $\vec{v}(t)$ as seen from S, and $\vec{u}(t)$ as seen from R. If R rotates with angular velocity $\vec{\omega}$ relative to S, show that $\vec{v} = \vec{u} + \vec{\omega}\times\vec{r}$ [use the result of 2(a)]. [6]

Group – C

5. a) What do you mean by proper length, proper time and an event? [0·5+0·5+1]

   b) Write down the Lorentz transformation equations and hence derive the equations of relativistic addition of velocities. [1+4]

   c) A particle of rest mass m_0 is travelling so that its total energy is just twice the rest mass energy. It collides with a stationary particle of rest mass m_0 to form a new particle. What is the rest mass of the new particle? [3]

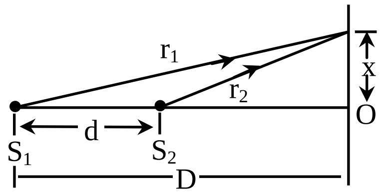
6. a) Write down the two fundamental postulates of Special Theory of Relativity. [1+1]

   b) Derive the formula for relativistic variation of mass with velocity. [5]

   c) One astronaut returns on earth after one year of interstellar travelling, having moved with a velocity half of the speed of light. How much younger he will appear after this journey? [3]

Group – D

7. a) Two similar coherent point sources S_1 and S_2 are separated by a distance 'd' and the interference pattern is observed on a screen kept at a distance 'D' from S_1 along the line joining the sources as shown in fig :



   i) Can you guess the shape of the fringe? [1]

   ii) Find the distance x of the n^{th} bright fringe from the point O. [4]

   b) Show that the dark and bright fringes produced in Young's experiment are equally spaced. [5]

8. a) A biprism of obtuse angle 176° is made of glass of refractive index 1·5. A slit illuminated by a monochromatic light is placed 20 cm behind the biprisim and the width of the interference fringes formed on a screen 80 cm in front of the biprism is found to be 0·0325 mm. Find the wavelength of light. [5]

   b) What is Fermat's principle? [1]

   c) How laws of refraction is proved by Fermat's principle? [2]

   d) How achromatic doublet reduce chromatic aberrations. [2]

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